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CONCENTRATION FIELDS IN DUSTY GAS FLOWS

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Abstract—It is well-known that particle inertia in dusty gas flows may cause a certain complex behavior of the concentration fields, such as the enrichment of particle concentration along particle trajectories. Traditional analyses of these and similar problems were made by treating the gas and the particles as two interpenetrating continua and using the two-phase flow equations for the description of the two phases. For cases with relatively low Stokes number, the concentration field may be obtained by a regular perturbation analysis of the equations of change using the Stokes number as a parameter. In the present study, we adopted the Lagrangian view of particle motion and obtained particle concentrations by applying mass balance to neighboring trajectories. In contrast to the earlier works, this approach can be used for all Stokes numbers and its application (at least in the cases considered) is straightforward. To demonstrate its utility, we present the results of our analyses of two specific problems and compare our results with those obtained by previous investigators.

Key Words: aerosols, collection efficiency, inertial effects, dusty gas flow, two phase flow, Lagrangian method

INTRODUCTION

The dynamics of dusty gases represents an important subject of study because of its relevance to several important natural and industrial processes. In analyzing dusty gas flows, one is often interested in the concentration field within the dust flow or the stability of the flow. A review by Marble (1970) provides an overview of these types of problems and their applications.

Dusty gas flow has been examined by considering the gas and the particles to constitute two continua. The gas motion is given by the Navier–Stokes equations of motion, while the particle motion is described by a separate set of equations of change for mass and momentum with the assumption that the presence of the particles does not alter the background clean gas motion significantly. Such two-phase models have been used to study the stability of plane parallel flows (e.g. Saffman 1962) and to obtain concentration field information under various conditions (e.g. Michael 1968; Michael & Norey 1970; Fernandez de la Mora & Rosner 1981, 1982; Ishii *et al.* 1992).

Both the earlier works of Michael (1968) and Michael & Norey (1970) as well as the more recent ones of Fernandez de la Mora & Rosner (1981, 1982) have shown that the particle concentration field may become nonuniform if the inertial effect is sufficiently large. For example, for creeping flow past a sphere (and also potential flow about a cylinder), Fernandez de la Mora & Rosner (1982) found particle concentration enrichment along the stagnation streamline. With Brownian particles this concentration enrichment phenomenon may become important, since in combination with the Brownian diffusive effect it may lead to significantly enhanced particle deposition rates. Inertial enrichment of particle concentration was also found in the stagnation point flow case by Peters & Cooper (1991) and by Ramarao & Tien (1993).

For the present study, we propose a different approach for determining the motion of the dusty gas and the resultant concentration field by adopting the Lagrangian description of particle motion. From the solution of the equations of particle motion, particle trajectories may be obtained. Assuming that there are no concentration singularities within the flow, we can obtain particle deposition flux at the collector surface as well as particle concentration within the flow by applying the mass balance requirement over neighboring trajectories. The major work involved in applying this method consists of the determination of the particle trajectories—a task that can be accomplished by a numerical integration of the Lagrangian equations of motion.

The results obtained by applying this method to two specific problems, the flow past a sphere under creeping flow and potential flow conditions, show that the concentration field has a complex behavior and that the inertial enrichment in concentration is found in the upstream portions of the flow. In the downstream portion, particle concentration may be depleted and may be significant near the rear stagnation region. Good agreement was found between our results and those reported by Michael & Norey (1970) previously for the low Stokes number case.

In the limiting situation of vanishing particle inertia, it can be shown that the particle concentration remains uniform throughout the flow if the initial condition is uniform. In the other limit of infinitely high inertia, application of the Lagrangian method indicates that the particle concentration remains uniform throughout the flow. Thus, particle concentration undergoes a maximum as a function of the Stokes number. Investigations using the earlier theory of Fernandez de la Mora & Rosner (1982), along the front stagnation line also reveal this feature.

DETERMINATION OF THE CONCENTRATION FIELD IN A FLOW

Consider a general fluid flow denoted by the velocity field $\mathbf{u}(\mathbf{x})$. Let us suppose that particles enter this flow field at a location far from the region of immediate interest with a veloctiy $\mathbf{v}_0(\mathbf{x}_0)$. Figure 1 shows an example axisymmetric flow field.

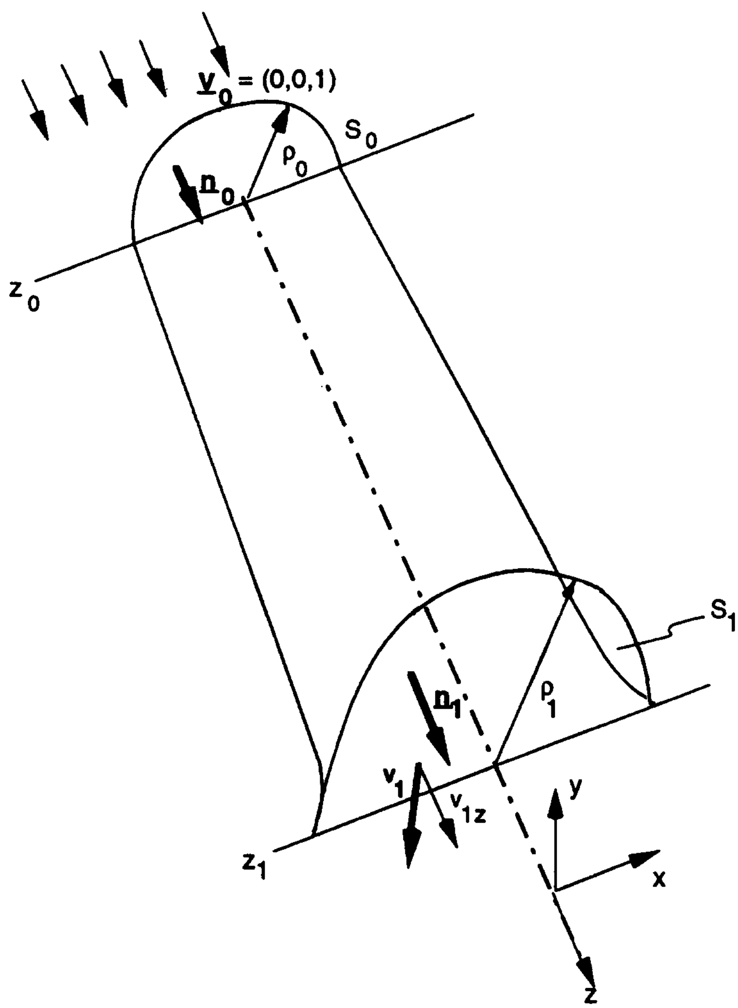

Figure 1. Definition sketch—general axisymmetric flow.

Axisymmetric flow field

Analogous to streamtubes for fluid flow, one may consider a particle streamtube with its surface constructed from a bundle of particle trajectories, as shown in figure 1. We consider two planes, S_0 and S_1 corresponding to two cross sections of the tube, located over a certain distance. The flux of particles entering the tube through S_0 at location 0 is given by

$$j_0^* = \int_{S_0} c_0^* \mathbf{v}_0^* \cdot \mathbf{n}_0 \, \mathrm{d}A \qquad [1]$$

and the flux of particles leaving the tube through S_1 is given by

$$j_1^* = \int_{S_1} c_1^* \mathbf{v}_1^* \cdot \mathbf{n}_1 \, \mathrm{d}A \qquad [2]$$

where c is the concentration of particles, v is the particle velocity and $\mathbf{n}_0$ and $\mathbf{n}_1$ are the unit normal vectors over the differential area $\mathrm{d}A$ over S_0 and S_1; the * indicates dimensional quantities and the subscripts 0 and 1 relate, respectively, to the initial conditions (at entry location) or to any other location in the flowfield. If we assume that particles do not cross the streamtube and, furthermore, that there are no singularities in particle concentration within the streamtube, these two fluxes should be equal. Since the surfaces S_0 and S_1 are perpendicular to the z-axis, their unit normal vectors $\mathbf{n}_0$ and $\mathbf{n}_1$ are parallel to the z-components of the particle velocity vectors $\mathbf{v}_0$ and $\mathbf{v}_1$ (and also $\mathbf{u}_0$ and $\mathbf{u}_1$). Using the particle streamtube bounded by S_0 and S_1 as the control volume:

$$c_0^* v_{0z}^* \, \mathrm{d}\rho_0^2 = c_1^* v_{1z}^* \, \mathrm{d}\rho_1^2, \qquad [3]$$

where ρ is the off-center distance and $\rho^2 = x^2 + y^2$. The variables are shown in figure 1.

We normalize the spatial variables by a length R, the concentrations by the inlet concentration (assumed to be uniform over S_0) and the velocities by the inlet z-directional velocity of the fluid U_0. In terms of these dimensionless variables, [3] becomes

$$c_1 = \frac{1}{v_{1z}} \frac{\rho_0}{\rho_1} \frac{\mathrm{d}\rho_0}{\mathrm{d}\rho_1}. \qquad [4]$$

Equation [3] indicates that to determine the concentration at any location $\mathbf{x}$ we need to determine v_{1z}, the particle velocity in the z-direction, and the relative increase in the off-center position of the particle to its initial off-center position, $\mathrm{d}\rho_0/\mathrm{d}\rho_1$.

For a given flow field, particle trajectories can be determined by applying Newton's law. Assuming that Stokes' law may be used to estimate the drag force on the particle due to the relative

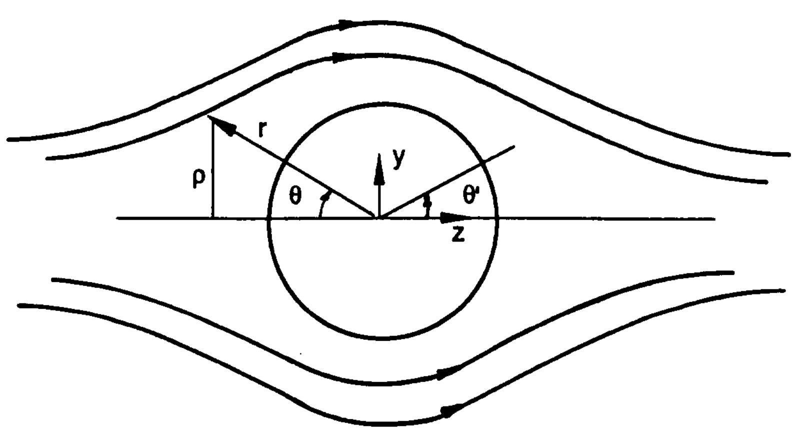

Figure 2. Definition sketch—flow over a sphere.

motion between the gas and the particle, the equation of particle motion (or trajectory equation) is given as

$$\mathrm{St}\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2}+\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}-\mathbf{u}=0, \qquad [5]$$

where St is the Stokes number and t is time.

The method outlined here is based on the Lagrangian description of particle motion. We will apply it to two flows to obtain the respective concentration fields. These flows have been investigated previously and various approximate analytical and numerical solutions are available.

CREEPING FLOW OVER A SPHERE

The flow field is given by the Stokes solution:

$$u_x=\frac{3}{4}\frac{xz}{r^3}\left(\frac{1}{r^2}-1\right), \qquad [6a]$$

$$u_y=\frac{3}{4}\frac{yz}{r^3}\left(\frac{1}{r^3}-1\right) \qquad [6b]$$

and

$$u_z=1-\left(\frac{3}{4r}+\frac{1}{4r^3}\right)+\frac{3z^2}{4r^3}\left(\frac{1}{r^2}-1\right), \qquad [6c]$$

where r is the radial distance in spherical coordinates ($r^2=x^2+y^2+z^2$).

In streamfunction form, the flow field is given by

$$\Psi=-\tfrac{1}{2}r^2\sin^2\theta'\left(1-\frac{3}{2r}+\frac{1}{2r^3}\right); \qquad [7]$$

θ' is the angle from the rear stagnation point.

Based on the fluid velocity expressions of [6a–c] and specific initial positions, y_0, particle trajectories can be determined from [5] and the concentration along the trajectory from [4]. Table 1 gives the results of a number of sample calculations, namely values of the dimensionless concentration at various positions (z) along particle trajectories. Notice that the concentration increases initially along each particle trajectory until it reaches a maximum. It then decreases as the trajectory goes around the sphere and then increases again in the rear half of the flow. If the initial position of the trajectory is sufficiently close to the front stagnation line, however (in this instance, if $y_0<0.5$), the concentration never recovers to the equilibrium value of 1. The results presented in tables 2–5 show the local concentration values corresponding to different St values. As can be expected, at high inertia, represented by high St values, differences in particle concentrations within the flow field become accentuated.

COMPARISON WITH EARLIER SOLUTIONS

The perturbation solution of Michael & Norey (1970)

Michael & Norey (1970) presented a perturbation solution of the problem considered above which is valid for low particle inertia. A regular perturbation solution was found with the particle and fluid velocity fields being given by the classical Stokes flow field about a sphere at the leading order. The particle concentration was expressed as

$$f=f^0+f^1+\ldots, \qquad [8]$$

where f is the particle mass concentration which to the leading order is unity. The concentration f is related to c as $c=f/f^0$. The first-order perturbation for the particle concentration, f^1, can be determined from the Stokes velocity field [see Michael & Norey (1970) for more details]. Along the front and rear stagnation line, the particle concentration can be expressed by the following simple relations:

front stagnation line,

$$f^1 = \frac{27}{8}\mathrm{St}\left[8\ln\left(1+\frac{1}{2r}\right)-\frac{4}{r}+\frac{1}{r^2}+\frac{1}{2r^4}\right]; \tag{9}$$

and

rear stagnation line,

$$f^1 = \frac{9\,\mathrm{St}}{8}\left[-24\ln\left(1+\frac{1}{2r}\right)+\frac{12}{r}-\frac{3}{r^2}-\frac{3}{2r^4}+48\ln\frac{3}{2}-\frac{61}{3}\right]. \tag{10}$$

For other positions within the flow, f^1 can be determined by integrating the following equation along a given fluid streamline:

$$\left(\frac{\partial f^1}{\partial \theta'}\right)_{\Psi=k} = \frac{2\sqrt{2}(1+r)[4k^2r(2r^2-1)-3(r-1)^3(r+1)(2r+1)]}{r^{9/2}k(r-1)(2r+1)^{1/2}(4r^2+r+1)}. \tag{11}$$

Since the present method determines particle concentrations along a given particle trajectory, to obtain results which can be compared conveniently with those based on Michael & Norey's (1970) analysis, we used the following procedure. For a given location (r, θ) on a particle trajectory, the corresponding streamfunction value is found from [7]. By identifying the appropriate streamlines, integration of [11] along the streamline can be done in order to obtain the corresponding concentration value. The results obtained by applying this technique were compared with the results presented in table 1 of Michael & Norey's (1970) paper and were found to be in suitable agreement.

Table 1 shows a comparison of the concentration values predicted by the present method with those calculated according to Michael & Norey (1970). It is quite clear that at $\mathrm{St} = 0.05$, the agreement is excellent. The minor deviations can be attributed to numerical errors in the computations rather than to any fundamental disagreement between the two techniques. Similar agreement was also found for $\mathrm{St} = 0.01$ and 0.02. On the other hand, for higher St values ($\mathrm{St} = 0.1$ and 0.5), comparison of the results obtained by these two methods indicates larger discrepancies. Since Michael & Norey's (1970) analysis is based on perturbation expansions, their method can

Table 1. Particle concentration[a] in Stokes flow over a sphere-uniform flow at infinity

	Concentration c					
	$y_0 = 0.10$		$y_0 = 0.20$		$y_0 = 0.50$	
z	Lagrangian method	MN	Lagrangian method	MN	Lagrangian method	MN
−5	1.002	1.001	1.002	1.001	1.002	1.001
−2	1.014	1.011	1.012	1.011	1.009	1.008
−1.8	1.017	1.016	1.016	1.015	1.010	1.010
−1.6	1.024	1.023	1.022	1.021	1.014	1.012
−1.4	1.036	1.035	1.032	1.029	1.016	1.015
−1.2	1.056	1.052	1.040	1.038	1.018	1.016
−1.0	1.066	1.063	1.044	1.042	1.019	1.017
−0.8	1.063	1.059	1.044	1.040	1.017	1.016
−0.6	1.051	1.045	1.032	1.031	1.015	1.013
−0.4	1.030	1.027	1.025	1.020	1.011	1.009
−0.2	1.002	1.007	1.012	1.007	1.003	1.004
0	0.985	0.987	0.989	0.993	1.001	0.999
0.2	0.970	0.968	0.984	0.979	1.000	0.994
0.4	0.957	0.949	0.976	0.967	0.994	0.989
0.6	0.933	0.932	0.954	0.956	0.990	0.985

[a]Particle concentration c as a function of z coordinate along a particle trajectory calculated by this work and by Michael & Norey's (1970) method (MN). (z_0, y_0) represents the initial particle position far away from the collector, where $\mathbf{v}(\mathbf{x}_0) = \mathbf{u}(\mathbf{x}_0)$. $\mathrm{St} = 0.05$, $z_0 = -1000$, $\Delta y_0 = 10^{-10}$.

Table 2. Particle concentration in flow for $y_0 = 0.10$

	Concentration c			
	St = 0.5		St = 0.25	
z	Lagrangian method	MN	Lagrangian method	MN
−5	1.007	1.006	1.006	1.003
−2	1.069	1.111	1.048	1.055
−1.8	1.102	1.158	1.067	1.078
−1.2	1.604	1.569	1.290	1.271
−1.0	2.128	1.742	1.406	1.339
−0.8	2.089	1.669	1.388	1.310
−0.2	1.250	1.076	1.066	1.038
0	1.04	0.876	0.967	0.938
0.2	0.858	0.692	0.889	0.843
1.0	0.538	0.316	0.683	0.619
2.0	0.752	0.679	0.848	0.831
5.0	0.883	0.762	0.899	0.879

MN = Michael & Norey (1970).

Table 3. Particle concentration on the stagnation trajectory at two locations, variation with the initial position y_0

	Concentration c	
y_0	$z = -1.01$	$z = -1.1$
0.5	1.10	1.20
0.4	1.132	1.274
0.2	1.26	1.630
0.1	1.410	2.124
0.01	1.754	3.258
5×10^{-3}	1.801	—
1×10^{-3}	1.840	3.539
10^{-4}	1.857	—
10^{-6}	1.874	—
10^{-7}	1.874	—
FMR	1.87	3.56

St = 0.25. FMR = Fernandez de la Mora & Rosner (1982).

be expected to be valid only when St is sufficiently low. The relatively large disagreement shown in table 2 is therefore not surprising.

The solution of Fernandez de la Mora & Rosner (1982)

Fernandez de la Mora & Rosner (1982) also treated this problem of determining the particle concentration in the Stokes flow. Their approach began with the same equations, which subsequently were simplified to a set of three coupled ordinary differential equations for the particle concentration and velocity components in the vicinity of the stagnation line.

We calculated the particle concentration along the stagnation line using the Lagrangian method. Since the actual stagnation line ($y = 0$) ends in a concentration singularity, we used an initial position which was approximately close to the stagnation line. Table 3 shows the estimated concentration at $z = -1.01$ for various values of y_0. The corresponding concentration obtained by numerically integrating the equations of Fernandez de la Mora & Rosner (1982) is also shown. By choosing the initial position to be sufficiently small, we can obtain very good agreement with the results based on the equations obtained by Fernandez de la Mora & Rosner (1982).

Table 4 gives the comparisons of the concentration values estimated by the Lagrangian method with those obtained using Fernandez de la Mora & Rosner's equations for two different values of z, -1.1 and -1.01. The latter value was chosen by these authors to be the position where the diffusion boundary layer begins. The agreement was good.

Since this method is not limited to the stagnation trajectory, we can determine the particle concentration at any other location around the sphere for any arbitrary value of St. Table 5 gives the concentration values for a high St case (St = 1.0). It is interesting to note that the perturbation solution of Michael & Norey (1970) failed in this case, as evidenced by the fact that at certain z-values the concentration values were negative—a behavior that is certainly not physically possible.

Limiting cases of low and high inertia

In the limit of vanishing inertia, particle trajectories and fluid streamlines are coincident and the particle and fluid velocities are equal. The continuity equation for the particles becomes

$$\frac{\partial c}{\partial t} + (\mathbf{u} \cdot \nabla)c = 0. \qquad [12]$$

At steady state, the concentration gradient is orthogonal to the fluid velocity. Thus, the concentration is a constant along the fluid streamlines. If the upstream concentration is uniform, the particle concentration within the flow field is also uniform. The momentum equation for the particles represents the flow of a clean gas with a density equal to that of the dusty gas. The flow

Table 4. Particle concentration along the stagnation line

St	Concentration c, $z = -1.1$, Lagrangian method	FMR	$z = -1.01$, Lagrangian method	FMR
0.25	1.592	1.61	1.816	1.810
0.50	2.49	2.50	3.539	3.60
0.70	3.491	3.480	6.28	6.30
1.0	3.338	3.25	27.456	27.64
1.1	2.80	2.7225	82.38	82.12
1.2	2.40	2.338	19.01	18.99

FMR = Fernandez de la Mora & Rosner (1982).

Table 5. Particle concentration at a higher St (= 1.0) value

z	Concentration c, $y_0 = 0.1$, Lagrangian method	MN	$y_0 = 1.0$, Lagrangian method	MN
−5	1.009	1.0111	1.009	1.01
−2	1.078	1.222	1.045	1.09
−1.8	1.110	1.316	1.053	1.111
−1.2	1.7020	2.210	1.087	1.15
−1.0	7.580	2.876	1.10	1.128
−0.8	9.936	2.619	1.111	1.103
−0.2	2.994	1.1464	1.113	1.035
0	2.0673	0.6996	1.102	1.001
0.2	1.512	0.3052	1.084	0.970
0.8	0.7046	−0.392	1.1064	0.915
1.0	0.606	−0.386	0.996	0.909
5	1.717	0.414	0.966	0.995
10	1.768	0.4118	0.988	1.002

$z_0 = -1000$, $\Delta y_0 = 10^{-10}$.

of the dusty gas can be identified as the same as the flow of a corresponding clean gas with a higher Reynolds number (Michael 1968).

It is easy to show that [4] also gives the concentration $c_1 = 1$ in the limit of vanishing inertia. The fluid and particle velocities are equal ($v_z = u_z$). The streamfunction in an axisymmetric flow represents the flow rate through a surface such as S_0 extending up to the streamline ψ. By continuity, this flow rate is a constant all along the streamline. Since the axial velocity v_z determines the flow rate through the surfaces S_0 or S_1, it can be seen that $d\rho_0^2/d\rho_1^2$, and v_{1z} are exactly equal to the reciprocal of each other. Thus, c_1 is uniform along the streamlines. (For example, if $c_1 = 1$, c remains unity everywhere.) This behavior is valid for any general axisymmetric flow where particle trajectories coincide with fluid streamlines.

In the opposite limit, when particle inertia is extremely high, particle trajectories are rectilinear and the particle velocity is uniform and equals its initial value. The relative increase in the area of a particle streamtube, $d\rho_0^2/d\rho_1^2$, is unity. Thus, from [4], we see that the concentration at any location within the flow is also uniform.

Since the concentration field is uniform and equal to its initial concentration in both the limiting situations, i.e. $\mathrm{St} = 0$ and $\mathrm{St} \gg 1$, the concentration at each location must go through a maximum (or a minimum) at some value of St. This is indeed confirmed, as shown for the sample calculation results given in table 6. This table shows the concentration at a given position as a function of the St evaluated by the Lagrangian method. A similar prediction was also obtained from the equations of Fernandez de la Mora & Rosner (1982). The agreement is quite satisfactory.

Table 6. Particle concentration at $z = -1.1$ and -1.01 as a function of St, showing the maximum

St	Concentration c, $z = -1.1$, Lagrangian method	FMR	$z = -1.01$, Lagrangian method	FMR
0.1	1.1926	1.1920	1.278	1.280
0.25	1.562	1.5550	1.816	1.810
0.5	2.445	2.4342	3.539	3.60
1.0	3.259	3.252	27.46	27.63
2.0	1.398	1.393	1.565	1.559
3.0	1.186	1.184	1.229	1.224

FMR = Fernandez de la Mora & Rosner (1982).

POTENTIAL FLOW OVER A SPHERE

Another case of dusty gas flow over a sphere was considered by Michael (1968). The background clean gas was assumed to be in uniform potential flow over the sphere. Using the same technique as described earlier for the Stokes flow, namely a regular perturbation expansion for the particle equations of motion in Eulerian form, Michael obtained predictions of the concentration field and also the drag exerted on the sphere in this flow. We compared the predictions of the concentration fields by our method with the results obtained by Michael (1968).

The velocity potential for a uniform flow over a sphere is given by

$$\phi = \left(r + \frac{1}{2r^2}\right)\cos\theta \qquad [13]$$

The velocity field is given by

$$u_r = -\left(1 + \frac{1}{2r^2}\right)\cos\theta, \qquad [14]$$

and

$$u_\theta = -\left(1 + \frac{1}{2r^2}\right)\sin\theta. \qquad [15]$$

As in the earlier case, an analytical form for the concentration along the front stagnation line has been provided by Michael (1968). Table 7 shows the concentration along the front stagnation line determined by the Lagrangian method and the prediction using Michael's formula. Calculation results for three different values of St are shown in (St = 0.02, 0.1 and 0.5). As expected, the concentration values agree with Michael's prediction for St = 0.02. As St increases, the deviation between the calculations by the Lagrangian method and the perturbation method increases. The perturbation method significantly overpredicts the concentrations for St = 0.5. Table 8 shows the concentration calculations at three axial locations corresponding to the frontal, mid-plane and rear regions of the flow field ($z = -1.0$, 0.0 and 10.0) at three different St values. In general, the concentrations decrease as one moves away from the z-axis. However, at certain locations within the flow and corresponding to certain St values, the calculations are not possible because of intersecting particle trajectories. [Intersecting particle trajectories are found in other flows also e.g. Wang *et al.* 1986.] Notice also that in the mid-plane region around the sphere, i.e. $z = 0.0$, there is a steep concentration gradient along the y-coordinate. This is a consequence of the crowding of a large number of fluid streamlines and particle trajectories within this region.

The existence of a region of the flow where no particles are found was also predicted previously by Michael (1968). The particle trajectory which separates this particle-free region from the bulk of the flow may be evaluated in the following manner. Since this particle-free region is inaccessible to particles in the entrance region of the flow, one may determine the particle trajectory which separates this particle-free region from the rest of the flow field by integrating the particle's

Table 7. Particle concentration in potential flow over a sphere

	Concentration c					
	St = 0.02		St = 0.1		St = 0.5	
z	Lagrangian method	Michael (1968)	Lagrangian method	Michael (1968)	Lagrangian method	Michael (1968)
−2.0	1.0003	1.0003	1.0010	1.0017	1.0012	1.0083
−1.8	1.0008	1.0007	1.0020	1.0036	1.0022	1.0180
−1.5	1.0026	1.0027	1.0079	1.0144	1.0066	1.0718
−1.4	1.0044	1.0050	1.0131	1.0249	1.0104	1.1245
−1.3	1.0090	1.0092	1.0234	1.0461	1.0169	1.2306
−1.2	1.0172	1.0189	1.0486	1.0945	1.0296	1.4725
−1.1875	1.0199	1.0209	1.0538	1.1044	1.0325	1.5240
−1.1125	1.0375	1.0411	1.1140	1.2062	1.0542	2.0322
−1.1	1.0433	1.0469	1.1326	1.2352	1.0597	2.1778
−1.05	1.0853	1.0881	1.2842	1.4414	1.0899	3.2050

Table 8. Particle concentration at various locations for potential flow over a sphere

St = 0.02		St = 0.05		St = 0.1	
y	c	y	c	y	c
		$z = -1.0$			
0.1944	1.164	0.1788	1.462	0.1505	2.011
0.2827	1.102	0.2666	1.264	0.2389	1.498
0.4159	1.053	0.4010	1.126	0.3767	1.216
0.5264	1.031	0.5132	1.072	0.4916	1.118
0.6266	1.020	0.6148	1.044	0.5965	1.072
		$z = 0.0$			
1.0194	2.1820	1.0441	3.858	1.0787	—
1.0211	1.5976	1.0450	2.155	1.0786	—
1.0289	1.2106	1.0498	1.730	1.0799	4.875
1.0430	1.1480	1.0604	1.461	1.0850	2.533
1.064	1.0954	1.0777	1.272	1.0967	1.743
		$z = 10.0$			
0.3161	1.810	0.4638	4.808	0.6106	—
0.3231	1.575	0.4610	3.153	0.6107	—
0.3539	1.365	0.4790	2.105	0.613	6.313
0.4046	1.251	0.5064	1.687	0.623	3.119
0.4708	1.176	0.5491	1.457	0.645	2.143

equations of motion backwards in time using the initial condition, $z(0) = Z_0 \gg 1$ and $v_{0z} = 1$. The separating trajectory ends at the stagnation point instead of at a location far upstream.

CONCLUSION

When inertial effects are dominant in a dusty gas flow, the concentration of the particles deviates from unity and displays a complex behavior of enrichments and depletions in various regions of the flow. We demonstrate that this concentration field may be evaluated by a simple Lagrangian method based on a mass balance constructed over neighboring particle trajectories, assuming that particle trajectories do not intersect. We have applied this technique to the slow- and high-speed fluid motion over a sphere. At low St values, the concentration field predictions are in good agreement with the earlier perturbation solutions of Michael & Norey (1970) and Michael (1968). The advantages of the Lagrangian method are its simplicity (both conceptually and in computation) and its validity at all St values. In addition to the results reported above, the method was successfully applied to other types of flow [i.e. the ideal and viscous stagnation point flow given in Schlichting (1979) and potential flow over a cylinder] as well.

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